Study Guide

- Under which general condition might you choose to use a *for-loop* instead of a *while-loop* for computer programming? Explain under which conditions you might choose to use a *while-loop*?

- Solve the following utility maximization problem:

$$\max_{c_1,c_2,a_2} log(c_1) + \beta log(c_2) \tag{1}$$

$$\text{s.t. } (1+\tau)c_1 = \bar{a} - a_2 \tag{2}$$

$$\text{and } (1+\tau)c_2 = Ra_2, \tag{3}$$

where τ is the sales tax rate, $\bar{a}$ is initial wealth in period 1, R is the interest rate, and β is the discount factor.

  1. Find optimal savings and optimal consumption in each period as a function of the parameters. How does an increase in the sales tax rate affect optimal consumption?
  2. Solve for the indirect utility function. Either explain in words or show mathematically how a increase in the sales tax affects optimal utility.

- Explain the Bellman equation in detail. In your explanation, describe the different components and how they interact.

- Consider housing accumulation over the life-cycle. Let h_t be the agent's stock of housing at the beginning of period t, and let $u(c_t, h_t)$ denote per-period utility over non-housing and housing consumption, respectively. In order to change the housing stock, the agent incurs *closing costs* proportional to the change in the housing stock. In particular, closing costs of selling their current house are ρ_s% of the home's value, whereas buying their next home incurs proportional closing costs ρ_b% of the house they are buying. Total financial costs of buying and selling a house are $\rho_s h_t + \rho_b h_{t+1}$. Let wz_t be the agent's labor income in every period t, let a_{t+1} denote financial assets that the agent accumulates for the following period, and let R be the return on financial assets. Finally, suppose the agent discounts future utility at rate β and faces survival probability s_{t+1} in every period.

  1. How many state variables are in the agent's Bellman equation? What are the choice variables?
  2. Write the Bellman equation, including any constraints. *Hint: the only constraints are a budget constraint and no-Ponzi condition.*

- In 1972, Michael Grossman proposed a variation of the following dynamic model of health over the life-cycle. At the beginning of every period, agents have health stock h_t, which depreciates at rate δ_h. In order to improve health, the agent can pay financial costs f_t, which are translated into health outcomes according to medical technology $m(f_t, h_t, t)$. In other words, we can think of the ability of medical technology to improve health outcomes to depend on financial resources allocated to medical technology, current health, and age. Specifically, health evolves over the life-cycle according to:

$$h_{t+1} = h_t(1-\delta_h) + m(f_t, h_t, t). \tag{4}$$

Assume that the agent gets utility from consumption and health, so that per-period utility is $u(c_t, h_t)$. Suppose the agent saves a_{t+1} in financial assets with corresponding rate of return R, and let wz_t be the agent's labor income in period t.

1. Let β be the agent's discount factor, and suppose the agent faces survival probability s_{t+1} in every period t. Write the Bellman equation corresponding to this problem, and be sure to include any constraints.
2. Suppose now that survival probability is an increasing function of health, $s_{t+1}(h_{t+1})$. How would that change the Bellman equation?

- Let $\{s_{t+1}\}_1^T$ be a sequence of survival probabilities, and let ν be the population growth rate in a model economy.

1. Derive the population weights equation corresponding to a stationary distribution over ages, explaining each step carefully. Provide a general algorithm for solving these weights.
2. Let T be a lump-sum tax levied on every individual age 1 to $T_r - 1$. Further, suppose that every agent in the economy, regardless of age, receives a lump-sum benefit of b. Accounting for population weights, write out the government budget constraint.
3. Suppose that b is fixed. Describe an algorithm to solve for lump-sum tax T^* that clears the government budget constraint.