Optimal Capital and Labor Taxation in an Altruistic Economy

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June 26, 2012

Abstract

What is the optimal mix of distortionary capital and labor taxes in an altruistic economy? This question is answered by solving a dynamic general equilibrium model with production, in which finitely-lived individuals are linked intergenerationally through altruistic preferences. The government is tasked with financing an exogenous stream of government spending by levying distortionary capital and labor income taxes in a way that minimizes welfare loss in the economy. For a narrow set of parameter values, the numerical results show that nearly all government revenues should be raised through the labor income tax.

1 Introduction

This paper studies the Ramsey problem of optimal capital and labor taxation in an overlapping-dynasty model. While a vast literature has considered the welfare implications of tax rate changes in both an infinitely-lived agent model and a pure life-cycle model, none has solved the Ramsey taxation problem when different generations of finitely-lived individuals are linked through household altruism. This paper fills the void in the literature by incorporating intergenerational altruism into the pure life-cycle model and solving for the optimal capital and labor taxes.

Agent behavior in dynamic general equilibrium models is generally determined by either an infinitely-lived representative agent or a finitely-lived individual with life-cycle productivity variation. Each approach has proven to be a useful model, depending on the question being asked.[1] When the question regards the welfare effects of taxation, however, each approach can lead to quite different results. Chamley (1986) and Judd (1985), for example, each show that the optimal steady-state capital tax in an infinitely-lived agent model is zero.[2] Erosa and Gervais (2001) and Krueger, et. al (2009), however, show that the optimal capital tax can be significantly positive in a finitely-lived overlapping generations model. Fuster, et. al (2008) combine the two environments by studying a dynastic household of finitely-lived individuals and show that social welfare can be improved by eliminating the capital tax. This paper takes the analysis one step further by characterizing the welfare function over the entire set of capital and labor taxes that clear the government budget constraint in the dynastic household environment.

The importance of accounting for intergenerational altruism in economic models has been well documented. Kotlikoff and Summers (1981) study the role of intergenerational transfers in aggregate capital accumulation. They estimate less than 20% of aggregate capital is life-cycle wealth. Gale and Scholz (1994) study detailed household financial data from the Survey of Consumer Finances and find that intentional *intervivos* transfers (transfers between living people) account for at least 20% of U.S. wealth. After they add bequests, this estimate rises to at least 51% of U.S. wealth. Laitner (1992) proposes a model of life-cycle variation and intergenerational altruism that generates

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[1]Huggett (1993) and Huggett (1996), for example.

[2]See Atkinson, Chari, Kehoe (1999) for a survey of cases in which the optimal capital tax rate is zero.

a significant improvement in the U.S. aggregate wealth to GDP ratio, relative to a pure life-cycle model. Intergenerational altruism can improve the model's predictions at the individual level as well. Fuster, et. al (2008) find that the ratio of capital holdings of individuals 75 years or older to average capital holdings in this model is 1.75, while the same ratio in a pure life-cycle model is .52. They estimate that ratio in U.S. data to be 1.76.

The model economy studied in this paper assumes that consumption, saving, and leisure decisions are each made at the household level (Laitner (1992); Fuster, et. al (2007,2008)). Households are populated by an "older" generation and a "younger" generation of finitely-lived individuals. The dynastic structure of the household implies that decisions are made over the infinite horizon. Therefore, the household develops a stock of capital over time that fluctuates with the position of the household's members within the life-cycle. Because the value of capital to the household in this model is quite similar to the infinitely-lived agent, the resulting welfare effects of a capital tax are also similar. In other words, the model recommends a low capital tax for a narrow set of parameters. When the discount factor is reduced to another reasonable value, however, the model recommends a high capital tax.

This paper is organized as follows. The next section presents the model. Then, Section 3 discusses the calibration and presents the numerical results. Concluding remarks are offered in Section 4. The computational algorithm used to arrive at the numerical results is provided in the Appendix.

2   The Model

This section introduces the model economy. The model is a production economy with overlapping generations of dynastic households. Decisions regarding consumption, leisure, and savings are made at the household level. Firms maximize profit in a perfectly competitive environment by hiring labor and renting capital to produce units of output. Finally, the government must finance a stream of expenditures by taxing capital and labor in a way that minimizes social welfare loss.

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2.1 Households

Individuals are born into an infinitely-lived dynastic household but live finite lives of $2J$ periods. Households consist of a child and a parent. For the first J periods of an individual's life, he is a child. The parent then dies, a child is born into the household, and the individual becomes a parent. Each individual is endowed with a single unit of time in each period, which can be allocated between labor and leisure. Individuals' labor supply has age-dependent productivity z_j. Momentary utility of an age-j household is additive in the utility of each member of the household, and can be represented as:

$$U(c_j, l_j, c_{j+J}, l_{j+J}) = u(c_j, l_j) + u(c_{j+J}, l_{j+J}), \tag{1}$$

where c_j and l_j are consumption and leisure of the child, c_{j+J} and l_{j+J} are consumption and leisure of the parent, respectively, and $u(\cdot, \cdot)$ is strictly increasing, strictly concave, and twice continuously differentiable in each of its arguments. Individuals residing in the same household pool their resources, and decisions regarding consumption, labor, and savings are then made at the household level. The recursive formulation of the household problem can be written as follows the problem of the age-j household can be represented as follows:

$$V_j(k_j) = \max_{\{c_j, c_{j+J}, l_j, l_{j+J}, k_{j+1}\}} \{u(c_j, l_j) + u(c_{j+J}, l_{j+J}) + \beta V_{j+1}(k_{j+1})\} \tag{2}$$

$$s.t. \quad c_j + c_{j+J} + k_{j+1} = (1 + \hat{r})k_j + \hat{w}z_j(1 - l_j) + \hat{w}z_{j+J}(1 - l_{j+J}),$$

$$l_j, l_{j+J} \in [0, 1] \quad \text{and} \quad k_{j+1} \geq 0,$$

where k_j is the household's capital stock, β is the household's discount factor, and $\hat{r}$ and $\hat{w}$ are the after-tax interest rate and wage. The after-tax prices are defined as follows:

$$\hat{r} = (1 - \tau_k)r \tag{3}$$

$$\hat{w} = (1 - \tau_n)w, \tag{4}$$

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where r and w are the pre-tax interest rate and wage, respectively. The model economy is populated by J overlapping generations of households. Each staggered by one period so that an age-j household has an age-j and age-$j{+}J$ individual for $j = 1, \ldots, J$.

2.2 Firms

The firms in this economy rent capital at rate r and hire labor at rate w per unit in each period. Given constant returns to scale production technology F, the firm combines capital and labor inputs to produce a consumption good according to:

$$Y = F(K, L) = K^\alpha N^{1-\alpha}, \tag{5}$$

where

$$K = \sum_{j=1}^{J} k_j, \quad \text{and} \tag{6}$$

$$N = \sum_{j=1}^{J} [z_j(1 - l_j) + z_{j+J}(1 - l_{j+J})]. \tag{7}$$

Perfectly competitive firms take prices as given and maximize profits according to:

$$\max_{K,N} \ K^\alpha N^{1-\alpha} - rK - wN + (1 - \delta)K, \tag{8}$$

which lead to first-order conditions:

$$\alpha \left(\frac{K}{L}\right)^{\alpha-1} + (1 - \delta) = r \tag{9}$$

$$(1 - \alpha) \left(\frac{K}{L}\right)^{\alpha} = w. \tag{10}$$

2.3 Government

The objective of the government is to finance a sequence of expenditures in a way that minimizes the tax burden on households. The government fully commits to any policy. Tax instruments are limited to a linear capital tax and a linear labor income tax. A well know problem emerges when an upper bound does not exist on the capital tax rate. By levying an extremely high capital tax rate in the first period and purchasing a large stock of capital, the government can mimic a non-distortionary lump-sum tax and use the interest payments from the capital to finance the entire stream of expenditures. To make this an interesting and reasonable question, the government is limited to a capital tax rate of 100 percent. Of course, since individuals are restricted to non-negative capital holdings, a capital tax rate greater than 100 percent is not feasible. Suppose the stream of government expenditures in a steady-state is constant at an amount G, and let τ_k and τ_n be the capital and labor tax rates, respectively. Then the budget constraint of the government is:

$$\tau_k \tilde{K} + \tau_n \tilde{N} = G, \tag{11}$$

where $\tilde{K}$ and $\tilde{N}$ are the steady-state levels of aggregate capital and aggregate labor, respecitvely.

As mentioned, objective of the government is to finance a stream of expenditures in a way that minimizes the tax burden on households. Equivalently, the government chooses tax rates to maximize welfare while satisfying the government budget constraint. This problem can be addressed in two different ways. Much of the theoretical literature studies this question using the *primal* approach. The primal approach solves the government's problem by choosing over quantities, such as consumption, labor and capital. The tax rates are then implied by the solution to that problem. This paper takes the *dual* approach. The dual approach chooses over tax rates, and quantities are implied. This latter approach facilitates numerical analysis, so it is the approach taken in this paper.

Individuals' welfare is subjective, but the assumption made in this paper is that the government chooses to maximize an equally-weighted average of households' value functions. This choice of welfare function is consistent with the literature. The government's problem can now be stated as

follows:

$$\max_{\tau_k, \tau_n} \frac{1}{J} \sum_{j=1}^{J} V_j(k_j), \tag{12}$$

$$s.t. \quad \tau_k \tilde{K} + \tau_n \tilde{N} = G.$$

2.4 Competitive and Ramsey Equilibrium

The equilibrium concept in this economy is a stationary recursive competitive equilibrium. For the economy to be in a competitive equilibrium, individuals and firms choose optimally, prices clear markets, and the government's budget constraint is satisfied. The definition of a stationary recursive competitive equilibrium is formally stated in the context of the model as follows:

Definition 1. *Given fiscal policy* $\{G, \tau_k, \tau_n\}$, *stationary recursive competitive equilibrium is a set of value functions,* $\{V_j(k_j)\}_{t+1}^{J}$, *household decision rules,* $\{c_j(k_j), c_{j+J}(k_j), l_j(k_j), l_{j+J}(k_j), k_{j+1}(k_j)\}_{j=1}^{J}$ *and relative prices of labor and capital,* $\{w, r\}$ *such that:*

1. *Given fiscal policy and factor prices,* $\{c_j(k_j), c_{j+J}(k_j), l_j(k_j), l_{j+J}(k_j), k_{j+1}(k_j)\}_{j=1}^{J}$ *satisfy the household's decision problem* (2).

2. *Factor prices are competitive.*

3. *The government's budget constraint* (11) *is balanced.*

4. *Final goods and factor markets clear.*

Many different stationary competitive equilibria satisfy the government budget constraint. However, we are only interested in the competitive equilibrium that maximizes households' welfare. Since the government can fully commit, we categorize the solution as a Ramsey Equilibrium and define it formally as follows:

Definition 2. *A Ramsey Equilibrium is a stationary recursive competitive equilibrium that solves the government's problem* (12).

3 Quantitative Analysis

3.1 Functional Forms and Parametrization

The objective of the numerical analysis is to determine the optimal mix of capital and labor taxes when households have altruistic preferences. Erosa and Gervais (2002) show how the capital tax rate can be significantly positive when lives are finite, productivity varies throughout the life-cycle, and individuals within a household share no level of altruism. Therefore, to understand how their results compare to the altruistic version of the model in this paper, functional forms and parameters are set to those used in that paper. The functional forms are listed in Table 1:

Table 1: Functional Forms

	Function	Functional Form
Household Utility	$u(c_j, l_j)$	$\frac{1}{1-\sigma}(c_j^\gamma + \theta l_j^\gamma)^{\frac{1-\sigma}{\gamma}}$
Production	$F(K, L)$	$K^\alpha L^{1-\alpha}$
Government Spending	G	λY
Productivity Profile	z_j	$\exp(a_0 + a_1(j+1) + a_2(j+1)^2)$

Notice that government spending, G, is a constant fraction of output. Output, however, varies with changes in tax rates. Therefore, G is set to the fraction λ of output in an economy absent of taxes and remains constant throughout all tax experiments. The parameter values are listed in Table 2:

Table 2: Parameters

Parameter	Character	Value
Lifetime	$2J$	56
Discount Factor	β	.99
Coefficient of Relative Risk Aversion	σ	4
Consumption and Leisure Substitution Elasticity	γ	-.25
Labor Elasticity	θ	1.5
Government Spending fraction of Ouput	λ	.18
Depreciation Rate	δ	0
Capital Share of Ouput	α	.25
Productivity Parameters	a_0, a_1, a_2	4.47, 0.033, 0.00067

An important consideration is the additional intergenerational discount factor parameter in the life-cycle model that is not needed in the dynastic model (since $\beta < 1$ assures that the social welfare function is finite).[3]

3.2   Numerical Results

This subsection presents the results of the numerical analysis. The first part shows the decisions of the household while the second part characterizes the Ramsey equilibrium.

3.2.1   Household Policy Functions

The household allocates work time and chooses asset levels to maximize the discounted stream of utility over consumption and leisure. Figure 1 shows how the dynastic model generates a labor supply profile similar to the pure life-cycle model. Because a household pools resources, however, variation in total household labor income will be less than the individuals' labor income. Figure 2 shows total household labor income over the household's life-cycle. Unlike individuals who live for

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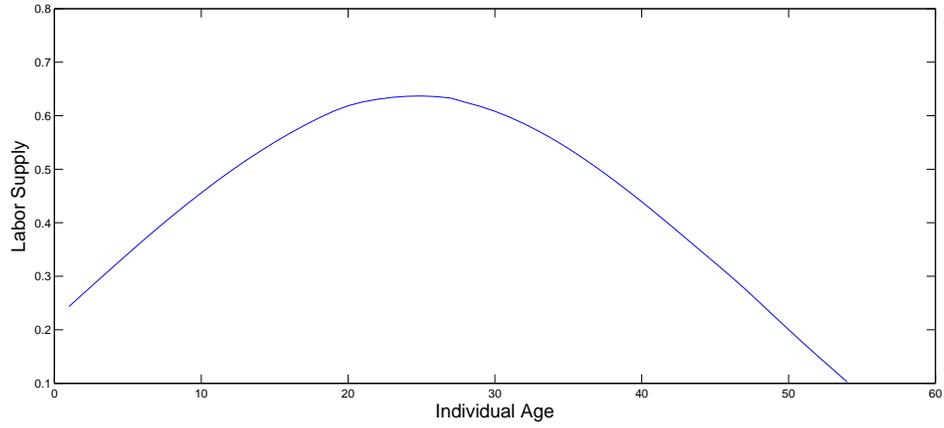[3]Equivalently, the value of this parameter is set to 1.

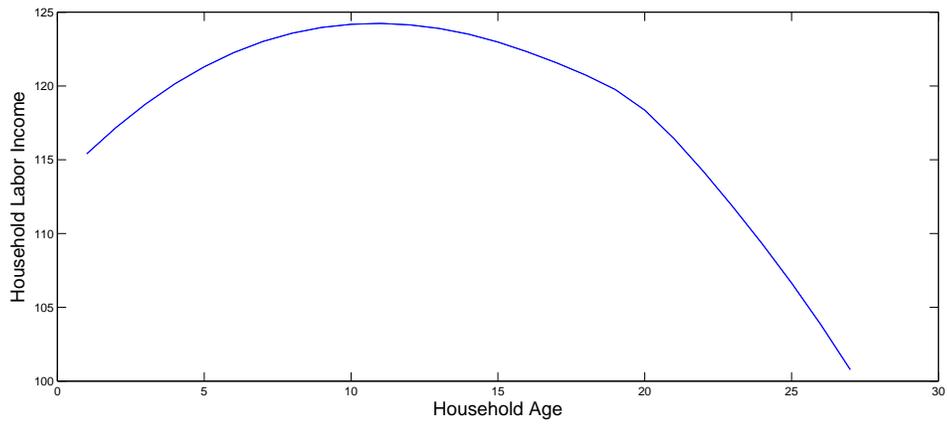Figure 1: Labor supply over the individual's lifetime.

Figure 2: Total household labor income over dynastic cycle.

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$2J$ periods, the household completes a cycle every J periods, then repeats itself. Figure 2 shows how household labor income is high early in the cycle, then falls towards the end. The household will choose asset levels to increase net worth and smooth consumption over this cycle, as shown in Figure 3. Notice how the household maintains a large stock of capital, and a relatively small portion of the capital stock varies with the fluctuations in the dynastic cycle.
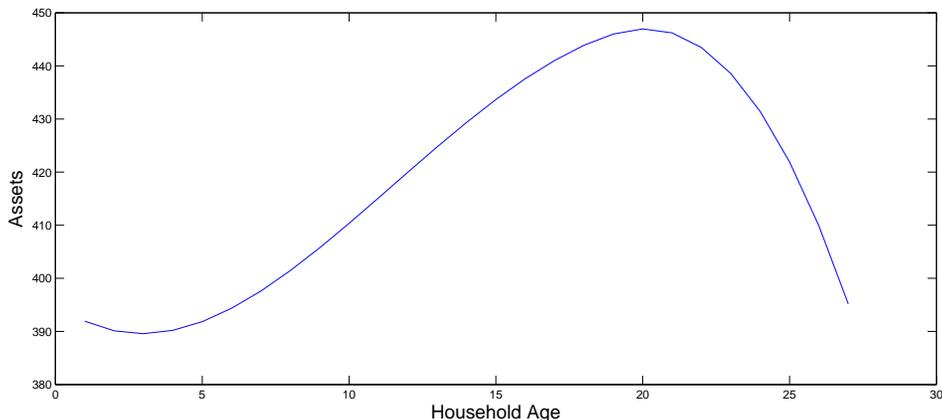

Figure 3: Asset choice over the dynastic cycle.

3.3   Optimal Fiscal Policy

To reiterate, the objective of the government is to choose the capital and labor tax rates in a stationary economy that maximize welfare and satisfy the government budget constraint. The bottom part of the figure shows the trade-off between capital and labor tax rates that clear the government budget constraint in a stationary equilibrium. The top part of the graph provides the welfare function at each of these capital and labor tax rates. The Ramsey Equilibrium is determined by the peak of this welfare function, where the labor tax rate is about 39% and the capital tax rate is about $\frac{1}{2}$%.

The welfare function peaks where labor tax rate is about 38 percent and the capital tax rate is about one-half percent.

To understand the intuition behind this optimal capital tax rate, consider the function of taxing
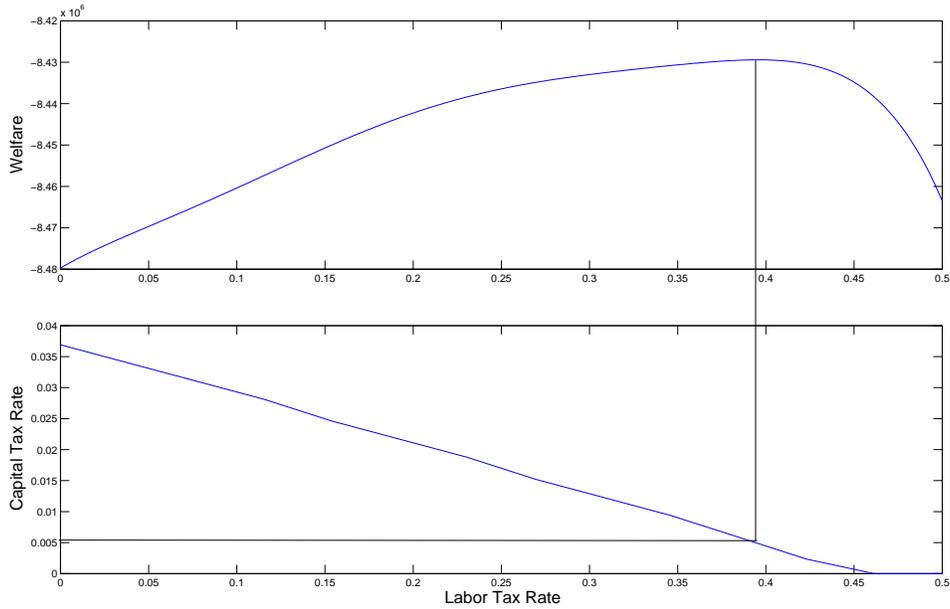
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Figure 4: Welfare tradeoff for various capital and labor tax rates that clear the government budget constraint in a steady state.

capital. Capital in this model functions as a method of partially redistributing income across households at different stages of the dynastic cycle. This component of the capital tax is analogous to the age-dependent tax proposed in Erosa and Gervais (2002) and Gervais (2009). The intuition behind the low capital tax follows from the very low benefits of redistribution in the dynastic model. Consider the asset choice of the household in figure 3. The variation in the household's capital stock as a percentage of average household assets over the dynastic cycle is quite low. Accordingly, the benefits of using the capital tax to redistribute income in this environment are small but positive.

Although the results of the this numerical exercise reinforce the intuition in the literature regarding the optimal taxation of capital, variations in the parameter values show that the results are not stable. Specifically, a reduction in the household discount factor from $\beta = .99$ to $\beta = .96$ implies that the entire amount of government expenditures should be financed by the capital tax. The direction of this shift in the optimal capital tax relative to the labor tax when the discount factor is reduced is consistent with both Erosa and Gervais (2002) and Fuster, et. al (2008). Erosa

and Gervais (2002) show that the optimal ratio of capital taxes to labor taxes rises as the intergenerational discount factor is reduced. Similarly, Fuster, et. al (2008) show that the welfare gains from removing the capital tax fall when the discount factor is reduced. The effect of a reduction in the discount factor on the optimal capital tax rate made sense because a capital tax is essentially a tax on future consumption. Since a reduction in the discount factor reduces the value of future consumption, the welfare loss of a capital tax should also fall.

4 Conclusion

This paper contributes to the optimal capital taxation literature by numerically solving the Ramsey problem in a dynastic model. The model was parametrized similar to other models in the literature. The numerical results showed that the optimal capital tax rate was quite low given these parameter values. These results, however, depended significantly on the subjective discount factor. Specifically, a reasonable change in the discount factor led to the opposite result where government expenditures are entirely financed by the capital tax.

Because of the sensitivity of the results to the subjective discount rate, future research could entirely characterize the set of solutions for changes in this parameter. Future research could also expand the model to include idiosyncratic income shocks across households, as in Fuster, et. al (2008). The role for redistribution through the capital tax would likely play a larger role in that environment. Finally, the stochastic version of the model could be applied to study the role of household risk-sharing in the optimal provision of unemployment insurance.

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Appendix: Computational Algorithm

This section describes the computational algorithm used to arrive at the results of the numerical analysis. The method of solving for a steady-state follows Marcet, et. al (2002). The first part of the computational algorithm is to solve the household problem over a grid of prices (r,w) by method of value function iteration. Then the individual's problem can be used to solve the Ramsey Equilibrium as follows:

1. Create a grid of labor tax rates $\tau_{n,1}, \tau_{n,2}, \ldots, \tau_{n,N}$.[4]

2. For each labor tax, $\tau_{n,j}$, guess a capital tax rate, $\tau_{k,0}$.

3. Define $\rho \equiv \frac{K}{L}$, since prices will only depend on this ratio. Then make an initial guess ρ_0, and use the individual agent's problem to solve for aggregate levels K_1 and L_1.

4. Use a weight, ω, to update the equilibrium capital-to-labor ratio, $\rho_1 = \omega\rho_0 + (1-\omega)\frac{K_1}{L_1}$, and iterate to convergence. The resulting quantities will satisfy a the conditions for a steady-state competitive equilibrium.

5. Let (K^*, L^*) denote the resulting competitive equilibrium capital and labor stocks. These values will imply a level of government revenue: $\tilde{G}$. Then if $\tilde{G} < G$, choose $\tau_{k,1} > \tau_{k,0}$ (and vice versa) by method of consecutive bisection. Iterate on this process to convergence, and save the resulting welfare and capital tax.

6. Iterate over steps 2-4 for each labor tax. Then interpolate the welfare function (within the optimization routine) on the labor tax grid to find the value of the labor tax that maximizes welfare. Then use that labor tax to find the associated capital tax that clears the government budget constraint.

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[4]Equivalently, create a grid of capital tax rates and conduct the analogous exercise by searching for a labor tax.