Assignment 4

This assignment will be graded out 30 points possible. Your final assignment average, which is weighted 30% of your final average, will equal the average of all of your assignment grades. I encourage you to work independently, since these assignments are designed to help you prepare for the exam and final project. If you have any questions, please see me or the TA for help. This problem set is due on **Wednesday, October 18 at the beginning of class**. Please email your MATLAB code to the TA and submit any written portion to him in class.

The goal of this assignment is to find the optimal Social Security replacement ratio, as defined in the class notes. Consider the following Bellman equation corresponding to the Social Security model:

$$V_t(a_t) = \max_{n_t, a_{t+1}} \quad u(c_t, n_t) + s_{t+1}\beta V_{t+1}(a_{t+1}) \tag{1}$$

$$\text{s.t.} \;\; c_t = (1-\tau_{ss})wz_t n_t + Ra_t - a_{t+1} + \mathbb{1}_{t\geq T_r} ss, \tag{2}$$

$$0 \leq n_t \leq 1, \tag{3}$$

$$\text{and} \;\; a_{t+1} \geq 0 \tag{4}$$

Notice that the no-Ponzi condition has been replaced by the constraint limiting the agent to saving positive amounts in every period. Accordingly, you should discretize the asset grid over positive values such that the lower bound is some very small but positive number. Population weights can be determined according to:

$$\mu_{t+1} = \frac{s_{t+1}\mu_t}{1+\nu}. \tag{5}$$

The government budget constraint corresponding to the Social Security program is:

$$\sum_{t=1}^{T_r-1} \mu_t \tau_{ss} w z_t n_t = \sum_{t=T_r}^{T} \mu_t ss, \tag{6}$$

In this assignment, you will have to optimize over both future assets and labor supply. Simply define the optimization problem over a two-dimensional vector, and everything generalizes accordingly. You will also need to set productivity to zero for retirement ages. Define the parameters as follows: $\beta = 1.01$, $R = 1.04$, $w = 1$, $T_r = 45$ (real-world age 65), $T = 80$, $\nu = 0.012$, and the productivity and survival rate vectors are provided with the assignment. Let the utility function be:

$$u(c,n) = \frac{(c^\gamma(1-n)^{1-\gamma})^{1-\sigma}}{1-\sigma}, \tag{7}$$

where $\gamma = 0.35$ and $\sigma = 3$.

1. Solve for the optimal replacement ratio and corresponding tax rate that clears the government budget constraint. To do this, I recommend creating a function that solves the life-cycle model. The inputs should be the Social Security benefit and tax rate, and the output should be a structure containing the aggregates. Included in the aggregates should be total benefit outlays, total tax revenue, aggregate welfare, and any other aggregates that you will need. Then, it's just a matter of discretizing a grid of benefits, $\{ss^i\}_{i=1}^N$, and for each benefit ss^i, find the tax rate that satisfies the government budget constraint $\tau^*_{ss}(ss^i)$. In the last step, you will have a grid of benefits (*x values*: $\{ss^i\}_{i=1}^N$) and a grid of aggregate welfare corresponding to each balanced-budget benefit and tax pair (*y values*:

$\{W(ss^i)\}_{i=1}^N$)). Maximize interpolated welfare over benefit values to determine the optimal benefit. Finally, calculate the replacement ratio corresponding to the optimal benefit.

2. Now we want to measure the aggregate consequences of instituting the optimal Social Security program, relative to an economy with no Social Security. Solve the model again with no benefit and zero tax rate, and determine the percentage change in each of the following aggregate variables: consumption, savings, labor supply, and welfare.