Assignment 3

This assignment will be graded out 30 points possible. Your final assignment average, which is weighted 30% of your final average, will equal the average of all of your assignment grades. I encourage you to work independently, since these assignments are designed to help you prepare for the exam and final project. If you have any questions, please see me or the TA for help. This problem set is due on **Friday, September 28 at the beginning of class**. Please email your MATLAB code to the TA and submit any written portion to him in class.

In 1967, Yoram Ben-Porath proposed a model of human capital accumulation that provides a macroeconomic policy framework for a variety of issues, including education policy and wage inequality. The theory suggests that individuals earn lower income earlier in life because they spend more of their time learning - either in school or on the job. In particular, supppose that i_t denotes the amount of time allocated to learning or investing in human capital accumulation, so that $1-i$ represented time actually working (or, equivalently, *working effectively*). At any given point in time, the individual has human capital stock h_t, and future human grows according to:

$$h_{t+1} = h_t + A(h_t i_t)^{\alpha}. \tag{1}$$

The time that they spend learning improves future human capital, but it reduces current income. Specifically, labor income is $wh_t(1-i_t)$, where w is the wage or rental rate of human capital. Ignoring savings of financial capital, we can write the budget constraint as:

$$c_t = w_t h_t (1 - i_t) \tag{2}$$

Let $u(c_t) = \frac{c_t^{\gamma}}{\gamma}$ be the contemporaneous utility function, and let β be the agent's discount factor. Further, assume the agent lives a maximum of T periods.

1. Write down the Bellman equation for the individual, including the constraints. Explain in words how the Bellman equation that you wrote captures the modeled behavior that we wish to study.

2. (*Numerical*) Let $T = 80$, $w = 1$, $\beta = 0.96$, $\gamma = 0.5$, $A = 0.08$, and $\alpha = 0.90$. Solve for the value function over the agent's life-cycle at each point in the state space (you can choose the discretization, but I recommend choosing a small but positive lower bound). Also, determine the optimal human capital investment time over the state space.

3. (*Numerical*) Now assume that $h_1 = 1$. Solve for the optimal human capital time investment and consumption profiles over the life-cycle. Plot both of these profiles for each age.