Assignment 2

This assignment will be graded out 30 points possible. Your final assignment average, which is weighted 30% of your final average, will equal the average of all of your assignment grades. I encourage you to work independently, since these assignments are designed to help you prepare for the exam and final project. If you have any questions, please see me or the TA for help. This problem set is due on **Friday, September 22 at the beginning of class**. Please email your MATLAB code to the TA and submit any written portion to him in class.

1. Consider the utility maximization problem:

$$\max_{c,n} \frac{c^\gamma}{\gamma} - n \tag{1}$$

$$\text{s.t. } c = (1-\tau)wn - T \tag{2}$$

where τ is a proportional income tax and T is a lump-sum tax. Let $\gamma = \frac{1}{2}$ and $w = 1$, and solve for utility corresponding to $\tau = 0$ and $T = 0.1$. Then, let $T = 0$ and solve for the proportional tax rate τ^* that generates the same amount of government revenue as when $T = 0.1$. What is the percentage change in utility resulting from reforming the tax structure from lump-sum to proportional? Explain your findings in words. (*Hint: Use bisection to solve for* τ^*.)

2. Consider the two-period utility maximization problem:

$$V_1(a_1) = \max_{a_2} \frac{c_1^\gamma}{\gamma} + \beta\frac{c_2^\gamma}{\gamma} \tag{3}$$

$$\text{s.t. } c_1 = a_1 - a_2, \tag{4}$$

$$\text{and } c_2 = Ra_2, \tag{5}$$

where $\gamma = \frac{1}{2}$, $\beta = 0.99$, $R = 1.04$, and $a_1 = 1$.

(a) Start by solving the one-period problem for $V_2(a_2)$ over a grid of values for a_2. You can choose the lower bound, upper bound, and number of grid points. Then solve the two-period problem at $t = 1$ by interpolating over the continuation values $V_2(a_2)$ that you solved for in the one-period problem at $t = 2$.

To put it differently, solve $V_2(a_2)$ over a grid of values $\{a_2^1, a_2^2, \cdots, a_2^N\}$, where the superscript denotes the grid point, and N is your number of grid points. Notice that this gives you a set of values $\{V_2(a_2^1), V_2(a_2^2), \cdots, V_2(a_2^N)\}$, which is just equal to $\{u(Ra_2^1), u(Ra_2^2), \cdots, u(Ra_2^N)\}$, since optimal savings in period 2 is zero. Then, use this value function as the continuation value to solve two-period problem for $V_1(a_1)$ in $t = 1$.

(b) Now, use the same grid $\{a_2^i\}_{i=1}^N$ for the discretization of a_1. Solve the previous problem at each grid point a_1^i, and plot the relationship between your a_1^i values and the corresponding optimal savings $a_2^*(a_1^i)$. Also plot the relationship between the value function $V_1(a_1^i)$ and your a_1^i values. Describe both of these graphs in your own words.