Assignment 1

This assignment will be graded out 30 points possible. Your final assignment average, which is weighted 30% of your final average, will equal the average of all of your assignment grades. I encourage you to work independently, since these assignments are designed to help you prepare for the exam and final project. If you have any questions, please see me or the TA for help. This problem set is due on **Wednesday, September 13 at the beginning of class**. Please email your MATLAB code to the TA and submit the written portion to him in class.

1. Consider the utility maximization problem:

$$\max_{c,n} \ \frac{c^\gamma}{\gamma} - n \tag{1}$$

$$\text{s.t.} \ \ c = wn \tag{2}$$

   Suppose now that the government levies a tax of $\tau\%$ on labor income, so that after-tax labor income is $(1-\tau)wn$, and government receives tax revenue τwn.

   (a) (*Analytical*) Solve the utility maximization problem analytically (i.e., using pencil and paper) for any arbitrary w, γ, and τ.

   (b) (*Numerical*) Now, let $\gamma = \frac{1}{2}$, $w = 1$, and linearly space a grid of τ values between 0 and 0.99 with 15 points. For each value τ in the grid, solve the utility maximization problem, and determine government revenue at each point: $\tau_i w n_i^* \ \ \forall \ \ i \in \{1, \ldots, 15\}$. Plot the resulting government revenue as a function of the tax rate (i.e., plot the τ values on the x-axis and $\tau w n^*$ values on the y-axis).

2. Numerically determine the government-revenue-maximizing tax rate τ^* by interpolating over the tax rates and government revenues corresponding to (1). Try both linear and spline interpolation. Explain in words the significance of the tax rate that maximizes revenue.

3. The mathematical (exponential) constant e can be expressed as the solution to the limit: $lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$. Let $\epsilon = .001$ and $\mathcal{N}$ be the natural numbers $\{1, 2, 3, \ldots\}$. Numerically find the smallest $n \in \mathcal{N}$ such that $|\left(1+\frac{1}{n}\right)^n - e| < \epsilon$. Note that e can be produced in MATLAB by the exponential operator e^x evaluated at $x = 1$: $exp(1)$.